# Statistics

# Lecture 14

Feb 19-8:47 AM

Testing Population Mean $\mu$:

Case I: $\sigma$ Known

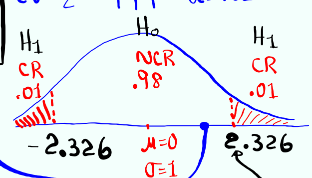
CV $Z$ invNorm

CTS $Z$
P-Value P → Z-Test

Given: $\boxed{H_0: \mu = 36}$
$\boxed{\text{claim is } H_0}$
$\boxed{\sigma = 12}$ $\alpha = .02$
$\boxed{\bar{x} = 40}$ $\boxed{n = 32}$
Test the claim.

$H_0: \mu = 36$ claim
$H_1: \mu \neq 36$ TTT

CV $Z$ TTT $\alpha = .02$




CV $Z = \text{invNorm}(.99, 0, 1)$

$\boxed{\text{CTS } Z = 1.886}$
$\boxed{\text{P-Value } p = .059}$

Z-Test
inpt: $\boxed{\text{Stats}}$
$\mu_0$: 36 $H_0$
$\sigma$: 12
$\bar{x} = 40$
$n = 32$
$\mu$ $\boxed{\neq \mu_0}$ $H_1$
$\boxed{\text{Calculate}}$

**CTS is in NCR**

**P-Value > $\alpha$**

$H_0$ valid $H_1$ invalid

Valid claim

**FTR the claim**

If we wish to reject the claim,
P-Value $\leq \alpha$ $.059 \leq \alpha$ Pick $\alpha$ to be .06, .07, .08, .09, .10

Dec 2-6:52 PM

College claims the mean age of all students is at most 29.5 Yrs. $\mu \le 29.5$ ← $H_0$

$\le$

I took a sample of 35 students, their mean age was 31.8 Yrs. $n=35$, $\bar{x}=31.8$

It is known that standard deviation of ages of all students is 8.5 Yrs. $\sigma=8.5$

No $\alpha \to .05$

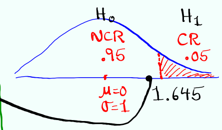
Test the claim.

$\sigma$ Known → Case I

$H_0$: $\mu \le 29.5$ claim

$H_1$: $\mu > 29.5$ RTT

CV $Z$ RTT $\alpha=.05$




CTS $Z=1.601$

P-Value $P=.055$

Z-Test

inpt: $\boxed{\text{Stats}}$

$\mu_0=29.5$ $H_0$

$\sigma=8.5$

$\bar{x}=31.8$

$n=35$

$\mu$ $\boxed{>\mu_0}$ $H_1$

$\boxed{\text{Calculate}}$

CV $Z=$ invNorm(.95,0,1)

CTS is in NCR

P-Value $> \alpha$

$H_0$ Valid $H_1$ invalid

valid claim

FTR the claim

If P-Value $\le \alpha$ $H_0$ invalid

$.055 \le \alpha$

choose $\alpha$ to be .06, .07, .08, ....

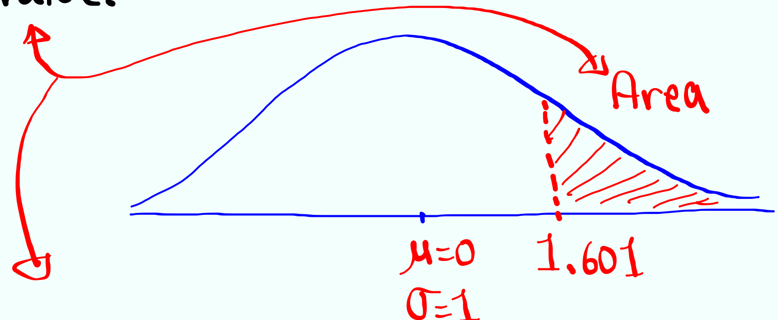
$H_0$ invalid → Invalid claim

Reject the claim.

Dec 2-7:07 PM

Given CTS $Z=1.601$ RTT

Find P-Value.




normalcdf(1.601, E99, 0, 1) $\approx \boxed{.055}$

Dec 2-7:25 PM

Case II: $\sigma$ Unknown

CV $t$ invT $df = n-1$

CTS $t$

P-Value $P$ ⇒ T-Test

Given $\boxed{H_0: \mu \geq 80}$

$\boxed{\text{claim is } H_1}$

$\bar{x} = 78$ $S = 10$

$\boxed{n = 15}$ $\boxed{\alpha = .01}$

Test the claim.

$\sigma$ unknown → Case II

CV $t$ LTT $\alpha = .01$

$df = n-1 = 14$

$H_0: \mu \geq 80$

$H_1: \mu < 80$ claim, LTT

CTS $t = -.775$

P-Value $P = .226$ ✓

T-Test

inpt: $\boxed{\text{Stats}}$

$\mu_0: 80$ $H_0$

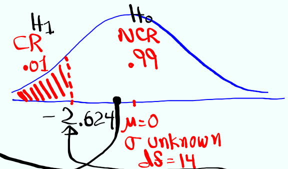
$\bar{x} = 78$

$S = 10$

$n = 15$

$\mu$ $\boxed{< \mu_0}$ $H_1$

$\boxed{\text{Calculate}}$




CV $t = \text{invT}(.01, 14)$

CTS is in NCR.

P-Value $> \alpha$

$H_0$ Valid

$H_1$ invalid

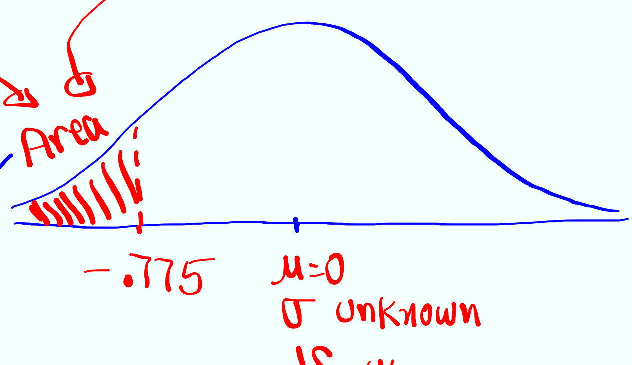
Invalid claim

**Reject the claim**

Dec 2-7:28 PM

Given CTS $t = -.775$ LTT $df = 14$

Find p-Value.




$\text{tcdf}(-E99, -.775, 14)$

$\approx \boxed{.226}$

Dec 2-7:42 PM

LA County health dept. claims that the mean age of all nurses in LA County is 48 Yrs.

$\mu = 48$ ← $H_0$

I took a Sample of 20 nurses, their mean age was 54 yrs with standard dev. of 6 Yrs.

No $\alpha \rightarrow .05$

Test the claim.

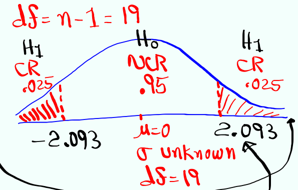
$\boxed{H_0: \mu = 48 \text{ claim} \quad H_1: \mu \neq 48 \text{ TTT}}$

$n = 20$
$\bar{x} = 54$
$S = 6$

$\sigma$ unknown → Case II

CV $t$ TTT $\alpha = .05$

$df = n - 1 = 19$




CV $t = \text{invT}(.975, 19)$

CTS $t = 4.472$
P-Value $P = 2.6 \times 10^{-4}$ ✓

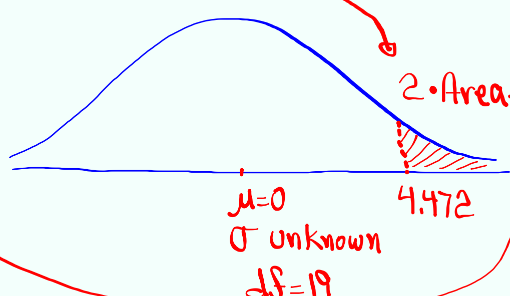
T-Test
inpt: Stats
$\mu_0: 48$ $H_0$
$\bar{x} = 54$
$S = 6$
$n = 20$
$\mu$ $\boxed{\neq \mu_0}$ $H_1$
Calculate

CTS is in CR
P-Value $\leq \alpha$
$H_0$ invalid $H_1$ valid
Invalid claim → Reject the claim

Dec 2-7:48 PM

Given CTS $t = 4.472$ TTT $df = 19$

Find p-Value.




$\text{P-Value} = 2 \cdot \text{tcdf}(4.472, E99, 19)$

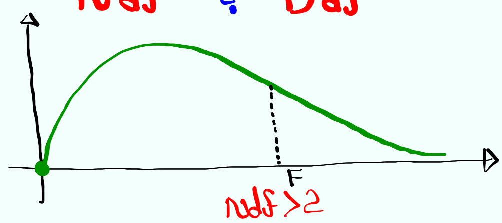
$= \boxed{2.6 \times 10^{-4}}$ ✓

SG 24 & 25

Dec 2-8:03 PM

F Dist.

1) Curve is not Symmetric.
2) Not bell-Shape
3) Total area = 1
4) It is positively Skewed.
5) It comes with two degrees of freedom **Ndf** & **Ddf**

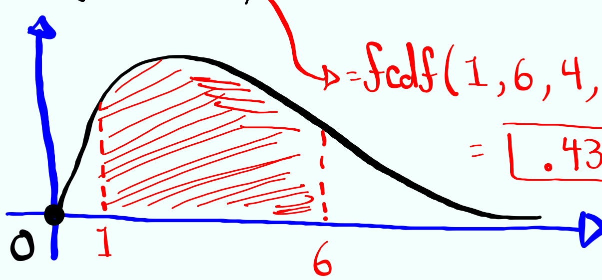



To find the area (probability), we use TI command

Fcdf(L, U, Ndf, Ddf)

Dec 2-8:20 PM

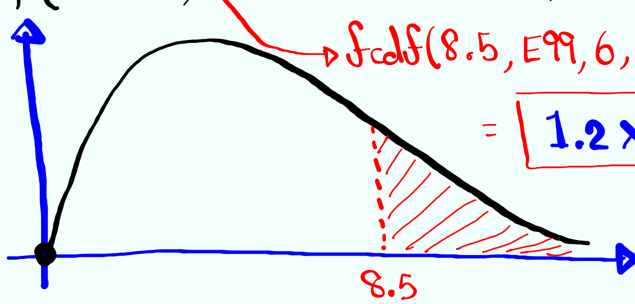
Find $P(1 < F < 6)$ with Ndf = 4 & Ddf = 12.




= fcdf(1, 6, 4, 12)

= $\boxed{.438}$

Find $P(F > 8.5)$ with Ndf = 6 & Ddf = 20.




fcdf(8.5, E99, 6, 20)

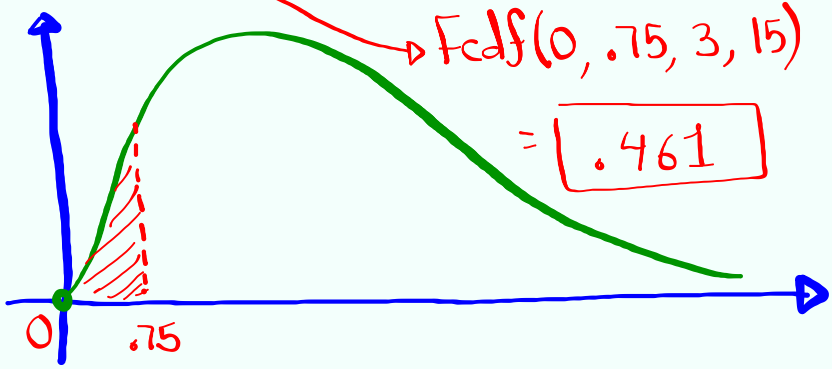
= $\boxed{1.2 \times 10^{-4}}$

Dec 2-8:25 PM

Find $P(F < .75)$ with $Ndf = 3$ & $Ddf = 15$.

$Fcdf(0, .75, 3, 15)$

$= \boxed{.461}$

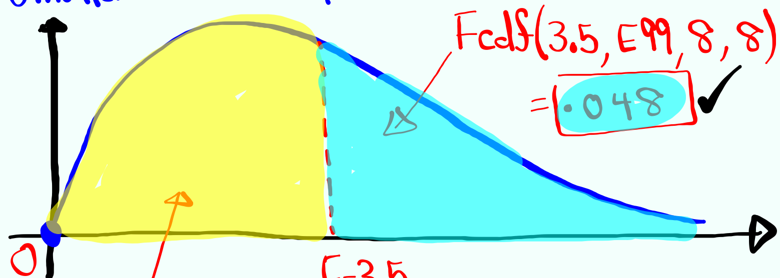



Dec 2-8:32 PM

Find the area on each side of $F = 3.5$ with $Ndf = 8$ & $Ddf = 8$, then multiply the smaller area by 2.

$Fcdf(3.5, E99, 8, 8)$

$= \boxed{.048}$ ✓




$Fcdf(0, 3.5, 8, 8) = \boxed{.952}$ ✓

$2 \cdot \text{Smaller Are} = 2(.048) = \boxed{.096}$

Dec 2-8:34 PM

Comparing two pop. standard deviations:

| | | |
|---|---|---|
| $H_0: \sigma_1 = \sigma_2$ | $H_0: \sigma_1 \geq \sigma_2$ | $H_0: \sigma_1 \leq \sigma_2$ |
| $H_1: \sigma_1 \neq \sigma_2$ | $H_1: \sigma_1 < \sigma_2$ | $H_1: \sigma_1 > \sigma_2$ |
| TTT | LTT | RTT |

| Sample 1 | Sample 2 |
|---|---|
| $n_1 =$ | $n_2 =$ |
| $S_1 =$ | $S_2 =$ |

1) $S_1 > S_2$

2) CTS $F = \frac{S_1^2}{S_2^2}$

3) $Ndf = n_1 - 1$, $Ddf = n_2 - 1$

CTS F

P-Value P

$\Rightarrow$ $\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{2-SampFTest}}$

inpt: $\boxed{\text{Stats}}$

Dec 2-8:40 PM

Use the chart below to test the claim that $\sigma_1 = \sigma_2$.

| Sample 1 | Sample 2 |
|---|---|
| $n_1 = 5$ | $n_2 = 8$ |
| $S_1 = 6$ | $S_2 = 5$ |

$H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

No $\alpha$ $\rightarrow$ use .05

$S_1 > S_2$ ✓

$Ndf = 5 - 1 = 4$, $Ddf = 8 - 1 = 7$

CTS $F = \frac{S_1^2}{S_2^2} = \frac{6^2}{5^2} = 1.44$

use 2-SampFTest

$\boxed{\begin{array}{l} \text{CTS } F = 1.44 \\ \text{P-Value } P = .631 \end{array}}$ $\rightarrow$ P-Value $> \alpha$ $H_0$ Valid $H_1$ invalid

Valid claim

FTR

Dec 2-8:46 PM

12 Female Students → S = 8 ✓

10 Male 〃 → S = 4

Use $\alpha = .02$ to test the claim that two Pop. standard deviations are not the Same.

| Females | Male |
|---|---|
| $n_1 = 12$ | $n_2 = 10$ |
| $S_1 = 8$ | $S_2 = 4$ |

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$ claim, TTT

$\alpha = .02$ Ndf = 11

$S_1 > S_2$ ✓ Ddf = 9

CTS $F = \dfrac{S_1^2}{S_2^2} = \dfrac{8^2}{4^2} = 4$ ✓

2-Samp F Test

$\boxed{\begin{array}{l}\text{CTS } F = 4 \checkmark \\ \text{P-Value } P = .047 \checkmark\end{array}}$

P-Value > $\alpha$

.047 .02

$H_0$ Valid

$H_1$ invalid Invalid claim

Reject the claim

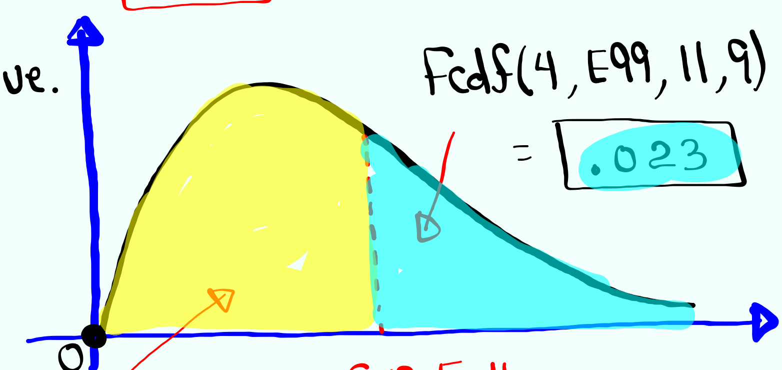
Dec 2-8:55 PM

CTS F = 4 $\boxed{\text{TTT}}$ Ndf = 11 , Ddf = 9

Find p-Value.

Fcdf(4, E99, 11, 9)

= $\boxed{.023}$




CTS F=4

Fcdf(0, 4, 11, 9) = $\boxed{.977}$

P-Value = 2 · Smaller area = 2(.023) = $\boxed{.046}$

Dec 2-9:02 PM